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Research on exciting current and induced current analytical model of EAF system

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Abstract

In this paper, large amount of subsection calculations and complex calculation of the traditional algorithm of an electronic anti-fouling system excitation coil current is analyzed, and a method based on a signal superposition theory of a linear system is put forward. What's more, a model system consisting of the electronic anti-fouling system and aqueous solution in the tube is established and the induced current is calculated carrying on reasonable approximates. Then the correctness of the proposed method and rationality of approximate system model are proved using finite element numerical method. This research will improve the adaptability of the electronic anti-fouling system in different water quality and optimize the effect on anti-scaling, and it provides a useful theoretical basis for further research on anti-scaling mechanism of electronic anti-fouling system.

Keywords: electronic anti-fouling system, anti-scaling, exciting current, excitation coil, induced current

1. Introduction

The electronic anti-fouling (EAF) system is a very useful electromagnetic field water treatment system for the scale problem. Through exerting high-frequency DC pulses on a solenoid coil winded in delivery pipe line of solenoid coil, an EAF system achieves anti-scaling performance. Domestic and foreign scholars generally thought that the excitation coil produces an alternating magnetic field, and the alternating magnetic field induces an alternating electric field in the hard water, which acted on scale ions [1-3]. So, it has an important significance for anti-scaling mechanism research to accurate calculation on

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current in the excitation coil changing with time and induced current in water. Traditional algorithm of the EAF system excitation coil current is using the subsection calculations. When the excitation coil current value on a specified time needs to be calculated, the current value of the end in every cycle before the specified time should be calculated, which means large amount of calculations. In this paper, the problem of the large amount of calculations of traditional algorithm will be solved by using a new method based on linear system signal superposition theory. Through this method, we calculate the coil current accurately with fewer calculation. On the system model which consists of the EAF system and aqueous solution in the tube based on reasonable approximate, current induced in water is calculated.

2. Analytically modeling

As shown in figure 1, an EAF system consists of a signal generator and an excitation coil. Changing magnetic field induces an annular current in an aqueous solution. So we can regard the water in a copper tube as a single turn coil. The equivalent circuit diagram of a system composing of processor and aqueous solution is shown in figure 2.

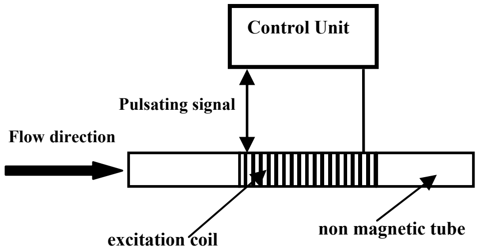


Figure 1. Variable-frequency EAF system structure diagram

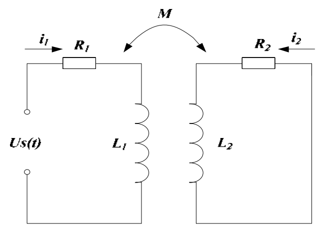


Figure 2. DC pulses water treatment system equivalent circuit diagram

Initial conditions are needed when a coil current is piecewise calculated using a traditional method. Because of the piecewise calculation, we need to calculate every discontinuous point as the initial conditions of next part current calculation which can make the computation increase greatly. This paper puts forward a new calculation method for coil current at any time. The main idea of this new method is to transform the pulse signal $u_s(t)$ into Fourier expansion. Thus we can get the expression of excitation coil current $i_1(t)$ under square wave pulsed excitation.

As shown in figure 2, i_1 represents magnetizing current in the coil, R_1 represents excitation coil resistance, L_1 represents coil inductance, i_2 represents the current in the water, R_2 represents water resistance， L_2 represents inductance of water， M represents mutual inductance and $u_s(t)$ represents excitation voltage signal

At any time points, the differential equations describing the circuit are:

$$\begin{cases} L_1 \dfrac{di_1}{dt} + R_1 i_1 - M \dfrac{di_2}{dt} = u_s(t) \quad (t \geq 0) \\ -M \dfrac{di_1}{dt} + L_2 \dfrac{di_2}{dt} + R_2 i_2 = 0 \end{cases}$$

(1)

As shown in the figure 3, the initial phase of excitation signal voltage is defined as zero.

Since $M = k\sqrt{L_1 L_2}$ and the excitation coil and the water equivalent coil are coaxial, we take k=1. Take the system composing of aqueous solution surrounded by the solenoid and the coil with 50 turns, length of 0.1m and radius of 0.02m for instance. Since the order of magnitude of excitation the coil inductance is 10^{-4}, the order of magnitude of the water equivalent inductance is 10^{-8} and the order of magnitude of the two coil mutual inductance value is 10^{-6}, then $L_2 << L_1$.

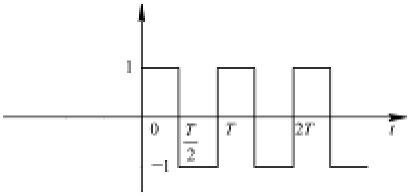


Figure 3 Square wave excitation signal added to the ends of the coil signal

For the large resistance of the hard water, the induced current in the water is less, and its rate is less also. Based on the above factors, the system equation (1) can be approximated as：

$$\begin{cases} L_1 \dfrac{di_1}{dt} + R_1 i_1 = u_s(t) \quad (t \geq 0) \\ -M \dfrac{di_1}{dt} + R_2 i_2 = 0 \end{cases}$$

(2)

From the first equation in equation (2), the current in a coil at any time points can be obtained. Then the induced current in water can be calculated by the second equation of equation (2).The widely adopted expression of coil current under the square-wave incentive is calculated by the following equations:

$$\begin{cases} L \dfrac{di}{dt} + Ri = U \quad (t \geq 0) \\ i(t_0) = i_0 \end{cases}$$

(3)

The *U* is segmental given. When the voltage of the signal is positive, *U* is positive pulse value, and when the voltage of the signal is negative, *U* is negative pulse value. Because the important content of electromagnetic anti-scaling mechanism research is the relationship between duration and scale inhibition effect, we should master the situation of coil current during a certain time and know current value at a sure time point. The problem by using equations (3) to calculate coil current is that we need initial conditions. Once there are a lot of cycles before a certain point, we need to calculate each unit step point of current value as the initial condition of the next step current calculation, which increases the computation greatly. It means that the whole calculation process will become very complex. Therefore, this paper proposes a calculation method of an exciting coil current.

In order to solve i_1, first, let square wave $u_s(t)$ signal do Fourier unfolding. The expression is:

$$u_s(t)=\frac{4A}{\pi}\left[\sin(\omega t)+\frac{1}{3}\sin(3\omega t)+\frac{1}{5}\sin(5\omega t)+...+\frac{1}{n}\sin(n\omega t)\right]$$

(4)

where A is amplitude of the square wave, ω is angular frequency of square wave. Note that the first equation of equation (3) is a linear differential equation, according to linear superposition principle and combining equation (4), equation (3) can be rewritten as:

$$\begin{cases}\begin{cases}L_1\dfrac{di_1^{(1)}}{dt}+R_1 i_1^{(1)}=\dfrac{4A}{\pi}\sin(\omega t)\\ i_1^{(1)}(0)=0\end{cases}\\ \begin{cases}L_1\dfrac{di_1^{(2)}}{dt}+R_1 i_1^{(2)}=\dfrac{4A}{\pi}\dfrac{1}{3}\sin(3\omega t)\\ i_1^{(2)}(0)=0\end{cases}\\ \dots\quad\dots\\ \begin{cases}L_1\dfrac{di_1^{(n)}}{dt}+R_1 i_1^{(n)}=\dfrac{4A}{\pi}\dfrac{1}{(2n-1)}\sin[(2n-1)\omega t]\\ i_1^{(n)}(0)=0\end{cases}\end{cases}\qquad (t\geq 0)\qquad (n=1,2,3,...,n)$$

(5)

The solutions of equation (5) at a certain time are $i_1^{(1)}(t)$, $i_2^{(2)}(t)$, …, $i_1^{(n)}(t)$, thus the solution of the first differential equations in equation (2) is:

$$i_1(t)=i_1^{(1)}(t)+i_1^{(2)}(t)+...+i_1^{(n)}(t)\qquad (n=1,2,3,...,n)$$

(6)

Then, at first, solve the first differential equations of equations (5):

$$\begin{cases}L_1\dfrac{di_1^{(1)}}{dt}+R_1 i_1^{(1)}=\dfrac{4A}{\pi}\sin(\omega t)\\ i_1^{(1)}(0)=0\end{cases}$$

(7)

The solution is:

$$i_1^{(1)}(t)=i_{Lh}^{(1)}(t)+i_{LP}^{(1)}(t) \tag{8}$$

where $i_{Lh}^{(1)}(t)$ is transient current , $i_{LP}^{(1)}(t)$ is steady-state current and its frequency is the same as incentive signal. The expressions of $i_{Lh}^{(1)}(t)$ and $i_{LP}^{(1)}(t)$ are shown in the (9) and (10).

$$i_{Lh}^{(1)}(t)=Ke^{-t/\tau}\qquad (\tau=\frac{L_1}{R_1}) \tag{9}$$

$$i_{LP}^{(1)}(t)=I_{Lm}\sin(\omega t) \tag{10}$$

Using a phasor method to calculate the stationary current circuit, it has the following transform:

$$\begin{cases}i_{LP}^{(1)}(t)\Rightarrow \dot{I}_{Lm}\\ u_s^{(1)}(t)=\dfrac{4A}{\pi}\sin(\omega t)\Rightarrow U_{Sm}\end{cases} \tag{11}$$

Circuit equations use a phasor method:

$$j\omega L_1 \dot{I}_{Lm} + R_1 \dot{I}_{Lm} = \dot{U}_{Sm} \tag{12}$$

$$\Rightarrow \quad \dot{I}_{Lm} = \frac{\dot{U}_{Sm}}{R_1 + j\omega L_1} = \frac{U_{Sm}}{\sqrt{R_1^{\,2} + (\omega L_1)^2}} \angle - tg^{-1} \frac{\omega L_1}{R_1} \tag{13}$$

$$\Rightarrow \quad i_{Lm}^{(1)}(t) = \frac{\frac{4A}{\pi}}{\sqrt{R_1^{\,2} + (\omega L_1)^2}} \sin(\omega t - tg^{-1} \frac{\omega L_1}{R_1}) \tag{14}$$

The solution of equations (7) is:

$$i_1^{(1)}(t) = \frac{\frac{4A}{\pi}}{\sqrt{R_1^{\,2} + (\omega L_1)^2}} \sin(\omega t - tg^{-1} \frac{\omega L_1}{R_1}) + Ke^{-t/\tau} \tag{15}$$

For $i_1^{(1)}(0) = 0$, constant K is calculated.

$$K = -\frac{\frac{4A}{\pi}}{\sqrt{R_1^{\,2} + (\omega L_1)^2}} \sin(-tg^{-1} \frac{\omega L_1}{R_1}) \tag{16}$$

thus：

$$\begin{aligned} i_1^{(1)}(t) &= i_{Lh}^{(1)}(t) + i_{LP}^{(1)}(t) \\ &= \frac{\frac{4A}{\pi}}{\sqrt{R_1^{\,2} + (\omega L_1)^2}} \sin(\omega t - tg^{-1} \frac{\omega L_1}{R_1}) - \frac{\frac{4A}{\pi}}{\sqrt{R_1^{\,2} + (\omega L_1)^2}} \sin(-tg^{-1} \frac{\omega L_1}{R_1}) e^{-t/\tau} \quad (t \geq 0) \end{aligned} \tag{17}$$

Similarly:

$$\begin{aligned} i_1^{(n)}(t) = {} & \frac{\frac{4A}{(2n-1)\pi}}{\sqrt{R_1^2 + [(2n-1)\omega L_1]^2}} \sin[(2n-1)\omega t - tg^{-1} \frac{(2n-1)\omega L_1}{R_1}] \\ & - \frac{\frac{4A}{(2n-1)\pi}}{\sqrt{R_1^2 + [(2n-1)\omega L_1]^2}} \sin[-tg^{-1} \frac{(2n-1)\omega L_1}{R_1}] e^{-t/\tau} \qquad (t \geq 0) \end{aligned} \tag{18}$$

According to the above results, the excitation coil current $i_1(t)$ under the square wave pulsed can be obtained finally.

In order to calculate an induced current in the aqueous solution $i_2(t)$, $\frac{di_1}{dt}$ should be obtained.

According to liner superposition, we can take the solution of $\frac{di_1^1}{dt}$ as example:

$$\frac{di_1^1}{dt} = \frac{\omega \frac{4A}{\pi}}{\sqrt{R_1^{\,2} + (\omega L_1)^2}} \cos(\omega t - tg^{-1} \frac{\omega L_1}{R_1}) + \frac{1}{\tau} \frac{\frac{4A}{\pi}}{\sqrt{R_1^{\,2} + (\omega L_1)^2}} \sin(-tg^{-1} \frac{\omega L_1}{R_1}) e^{-t/\tau} \quad (t \geq 0) \tag{19}$$

By analogy, finally:

$$\frac{di_1}{dt}=\frac{di_1^1}{dt}+\frac{di_1^2}{dt}+\frac{di_1^3}{dt}+\ldots+\frac{di_1^n}{dt} \qquad (n=1,2,3\ldots) \tag{20}$$

From the equation of $i_1(t)$, we can find that the coil resistance R_1 and the inductance L_1 play an important role. So the accuracy of these two values is very important.

$$R_1=\frac{2\pi rn\rho}{S} \tag{21}$$

where r is the average radius of excitation coil, n is the coil turns, ρ is the resistivity of coil wire materials, S is the cross-sectional area of the coil wire. According to the literature [4], the expression of L_1 is given as:

$$L_1=\mu_0 n^2\pi r^2 l\left[1-\frac{8}{3\pi}\frac{r}{l}+2\sum_{k=0}^{\infty}\frac{(-1)^k(2k-1)!!(2k+1)!!}{(2k+2)!!(2k+4)!!}\left(\frac{2r}{l}\right)^{2k+2}\right] \qquad (l>2r) \tag{22}$$

where μ_0 is vacuum magnetic conductance, n is turns of excitation coil of unit length , r is coil average radius, l is coil length.

3. Simulation and analysis

An excitation current in an actual coil is calculated and axial magnetic field intensity at several points on coil axis is then obtained using this current calculation method (called the analytic method). Through calculating $\frac{di_1}{dt}$, we get the induced current in the water. Since the better accuracy of finite element analysis method is widely adopted [5], several points' axial magnetic field intensity on coil axis and induced current in the water are calculated using the finite element numerical method to verify the correctness of the above calculation method and the rationality of the model result of approximation.

In our experiment, we choose 50 turns excitation coil with length l of 0.1 m, radius r of 0.02 m. The copper wire's resistivity is $1.7\times10^{-8}\,\Omega\cdot$m. The frequency of square wave pulse added at the coil is 500Hz and the amplitude is $\pm$ 1V. We choose to calculate the magnetic coil excitation and induced current in the water in a positive half-cycle of 0.001s. The half-cycle is evenly divided into 10 time points, which are 0.0001s, 0.0002s, 0.0003s, 0.0004s, 0.0005s, 0.0006s, 0.0007s, 0.0008s, 0.0009s and 0.001s. As the current expression using analytical methods is an infinite series, to minimize errors, the terms after the No. 10000 in the series are neglected.

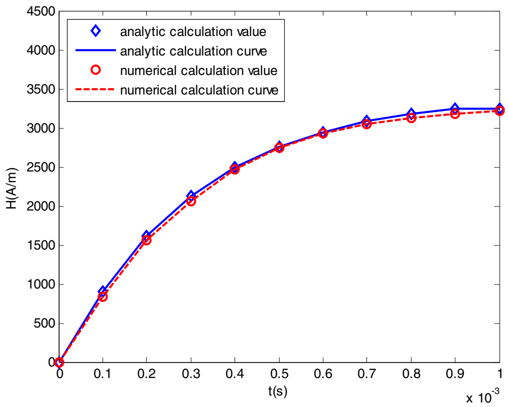
Finite element algorithm adopts the accurate 2D method to coil model. Take coil axis of symmetry center as origin, axis positive direction is the positive direction of z axis. Pick three points at Z axis: z = 0m, z = 0.01 m, z = 0.03 m, by analytical algorithm calculate current, we get three points in 10 different time points of axial magnetic field intensity. Formula is given as:

$$H_z=\frac{nI}{2}\left[\frac{(l+z)}{\sqrt{r^2+(l+z)^2}}+\frac{(l-z)}{\sqrt{r^2+(l-z)^2}}\right] \tag{23}$$

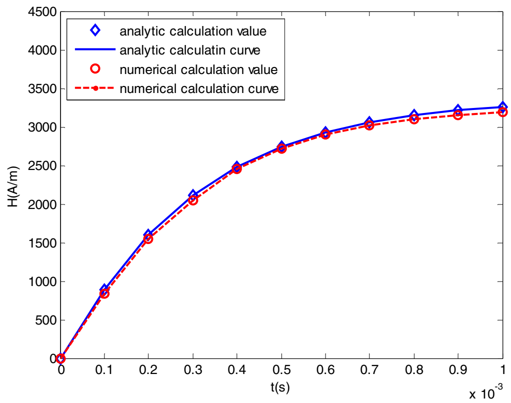
Table1. Results of magnetic field strength with two algorithms

	z=0m		z=0.01m		z=0.03m	
Time points（s）	Analytical algorithm calculated	Numerical calculation results	Analytical algorithm calculated	Numerical calculation results	Analytical algorithm calculated	Numerical calculation results

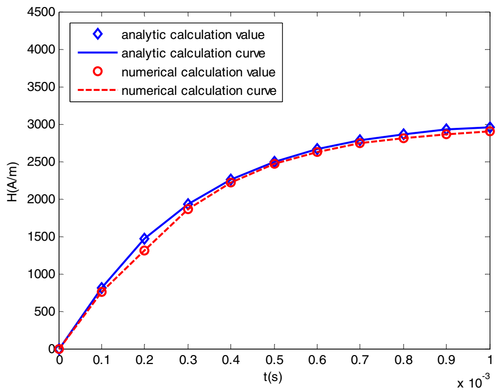
	results（A/m）	（A/m）	results（A/m）	（A/m）	results（A/m）	（A/m）
0.0001	898.7654	836.488	892.07	829.310	811.79	756.138
0.0002	1617.1	1553.33	1605.0	1542.18	1460.6	1308.26
0.0003	2131.3	2061.49	2115.4	2045.67	1925.1	1855.71
0.0004	2498.4	2468.44	2479.8	2449.51	2256.6	2222.05
0.0005	2760.4	2740.76	2739.8	2719.74	2493.3	2467.18
0.0006	2947.2	2922.99	2925.2	2900.56	2662.0	2631.22
0.0007	3080.3	3044.93	3057.4	3021.57	2782.2	2740.99
0.0008	3174.6	3126.53	3150.9	3102.54	2867.3	2814.44
0.0009	3238.8	3181.13	3214.7	3156.72	2925.4	2863.59
0.0010	3246.3	3217.67	3252.6	3192.98	2867.3	2814.44



a.z=0m



b.z=0.01m



c.z=0.03m

Figure 4. the fitted curve of two groups of the magnetic field strength

Table 2. Water induced current results using two algorithms at 10 time points

Time points（s）	Analytical algorithm calculated results （A）	Numerical calculation results （A）
0.0001s	-0.7164×10^{-3}	-0.6951×10^{-3}
0.0002s	-0.6626×10^{-3}	-0.6302×10^{-3}
0.0003s	-0.4737×10^{-3}	-0.4576×10^{-3}

0.0004s	-0.3553×10^{-3}	-0.3323×10^{-3}
0.0005s	-0.2539×10^{-3}	-0.2413×10^{-3}
0.0006s	-0.1795×10^{-3}	-0.1752×10^{-3}
0.0007s	-0.1297×10^{-3}	-0.1272×10^{-3}
0.0008s	-0.9366×10^{-4}	-0.9235×10^{-4}
0.0009s	-0.6828×10^{-4}	-0.6706×10^{-4}
0.0010s	-0.4992×10^{-4}	-0.4869×10^{-4}

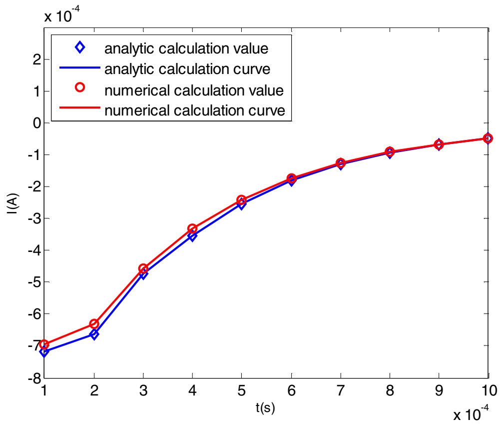


Figure 5. 10 points fitted curve at two groups of water induced current calculation results

According to the results obtained by these two methods, we can conclude that analytical algorithm results agree quite well with the numerical algorithm results. Thus the correctness of the analytical algorithm and the rationality of the model approximation are proved.

4. Conclusions

Innovations in this paper are to put forward a method based on a linear system signal superposition theory, and overcome calculation complicated problems in excitation coil current algorithm. What's more, the paper establishes a model system which consists of the EAF system and aqueous solution in the tube carrying on the reasonable approximate. Therefore the results of this paper provide a useful theoretical basis for further research on EAF system anti-scaling mechanism.

All calculations in this paper are being processed under the premise of constant conductivity of water. However, in the actual experiments, conductivity of the water will be slowly changed. It means that the water resistance will be changed. Therefore, the calculation method of the coil current and induced current method in this paper needs to be further improved.

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